BOOTHAM
SCHOOL
AGES 3-18

Introduction to A Level Maths

At Bootham we are able to offer both Mathematics and Further Mathematics at A Level.

To take A level Mathematics you must achieve a grade 7 or above at GCSE.
To take A level Further Mathematics you must achieve a grade 8 or 9.

This booklet gives you a taste of some topics covered in A level Mathematics and also the standard of work.

Working through the content of this booklet should help you understand whether A level Mathematics is for you.

Maths at Bootham School

We are delivering the Edexcel A Level Mathematics 9MAO specification and Further Mathematics 9FMD (2017) specification.

The courses are each a two year linear course. You can expect to be taught the Pure part of the course by one teacher and the Applied part by another.

Those studying Further Maths will have more lessons and will cover everything in the Pure part of A Level Mathematics before moving onto the Further Pure content. They will also be taught the Further Statistics, Mechanics and Decision Maths as appropriate in the second half of the course.

At Bootham we also cater for students who need to sit STEP, MAT and AEA papers for Oxbridge applications.

Pure Maths

To be successful at 'A' Level Maths it i the basics like simplifying and solving equations you need to be able to r quadratics.

Laws of indices

These 7 laws need learning as they will be used in nearly every section of Core Maths.

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

$$a^0 = 1$$

Examples

Simplify $6x^{\frac{3}{5}} \div 3x^{1\frac{2}{5}}$

$$= 2x^{\frac{3}{5}-\frac{7}{5}}$$

$$= 2x^{-\frac{4}{5}}$$

$$= \frac{2}{x^{\frac{4}{5}}}$$

$$= \frac{2}{\sqrt[5]{x^4}}$$

Evaluate $\left(2\frac{1}{4}\right)^{-\frac{3}{2}}$

$$= \left(\frac{9}{4}\right)^{-\frac{3}{2}}$$

$$= \left(\frac{4}{9}\right)^{\frac{3}{2}}$$

$$= \left(\sqrt{\frac{4}{9}}\right)^3$$

$$= \left(\frac{\sqrt{4}}{\sqrt{9}}\right)^3$$

$$= \left(\frac{\pm 2}{\pm 3}\right)^3$$

$$= \frac{\pm 8}{\pm 27} = \pm\frac{8}{27}$$

Surds

Surds are closely linked to indices because of the 5th and 6th law above. You need to be able to give an answer in **surd form** and **rationalise** surds.

Examples

Simplify $\sqrt{200} + \sqrt{18} - \sqrt{72}$

$$= \sqrt{100 \times 2} + \sqrt{9 \times 2} - \sqrt{36 \times 2}$$

$$= 10\sqrt{2} + 3\sqrt{2} - 6\sqrt{2}$$

$$= 7\sqrt{2}$$

Rationalise $\frac{3-\sqrt{5}}{4-\sqrt{5}}$

$$= \frac{3-\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}} = \frac{(3-\sqrt{5})(4+\sqrt{5})}{(4-\sqrt{5})(4+\sqrt{5})}$$

$$= \frac{12-4\sqrt{5}+3\sqrt{5}-5}{16-4\sqrt{5}+4\sqrt{5}-5} = \frac{7-\sqrt{5}}{11}$$

f Indices and simultaneous equations

s essential that you are able to easily manipulate algebra. As well as manipulate indices and solve simultaneous equations, including

Simultaneous Equations

You need to be confident using **elimination** and **substitution** for solving simultaneous equations. These techniques will be used in all of the modules.

Examples
Solve by elimination

$$3x-2y=-6$$
$$6x+3y=2$$

$$6x-4y=-12$$
$$6x+3y=2 \quad -$$
$$-7y=-14$$
$$\Rightarrow y=2$$
$$\Rightarrow 6x-8=-12$$
$$\Rightarrow 6x=-4$$
$$\Rightarrow x=-\frac{2}{3}$$

Solve by substitution

$$2y=2x-3$$
$$3y=x-1$$

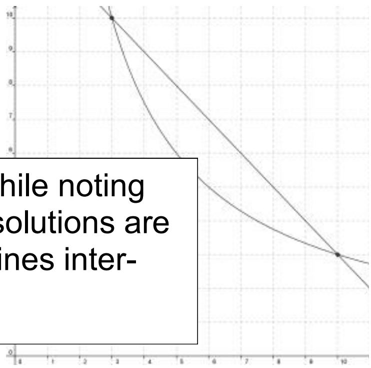
$$x=3y+1$$
$$\Rightarrow 2y=2(3y+1)-3$$
$$\Rightarrow 2y=6y+2-3$$
$$\Rightarrow 1=4y$$
$$\Rightarrow y=\frac{1}{4}$$
$$\Rightarrow x=3\left(\frac{1}{4}\right)+1=1\frac{3}{4}$$

You can also use substitution to solve simultaneous equations where one isn't linear.

Example Solve

$$x+y=13$$
$$xy=30$$

$$x=13-y$$
$$\Rightarrow y(13-y)=30$$
$$\Rightarrow 13y-y^2=30$$
$$\Rightarrow y^2-13y+30=0$$
$$\Rightarrow (y-10)(y-3)=0$$
$$\Rightarrow y=10, x=13-10=3$$
$$or\ y=3, x=13-3=10$$



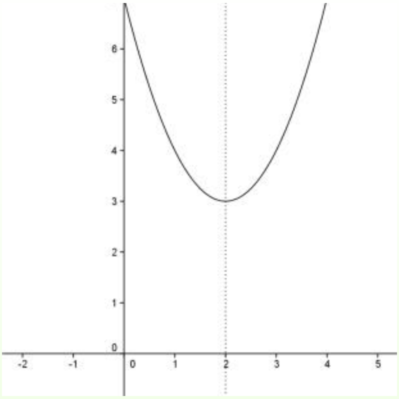
It is worthwhile noting that these solutions are where the lines intersect.

Pure Maths

Throughout the ‘A’ Level you will be ex
three algebraic methods and you need to b

Plotting Quadratics

A quadratic graph has a ∪ or ∩ shape and is symmetrical. To plot you must create a table of value, but to sketch a graph you may just pick out the key values such as the minimum and the intersections.



Factorising

This involves pulling out factors (ie putting the expression into brackets). You will have had plenty of practice of this before but now it is important that you can quickly factorise quadratics. Once in brackets you may solve the equation.

Examples
Solve:

$$x^2 - 4x - 12 = 0$$
$$\Rightarrow (x-6)(x+2) = 0$$
$$\Rightarrow x - 6 = 0$$
$$\Rightarrow x = 6$$
$$or\ x + 2 = 0$$
$$\Rightarrow x = -2$$

$$2x^2 = 8$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x^2 - 4 = 0$$
$$\Rightarrow (x-2)(x+2) = 0$$
$$\Rightarrow x = 2\ or\ x = -2$$

$$6x^2 - 7 = 11x$$
$$\Rightarrow 6x^2 - 11x - 7 = 0$$
$$\Rightarrow (3x-7)(2x+1) = 0$$
$$\Rightarrow 3x - 7 = 0$$
$$\Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$$
$$or\ 2x + 1 = 0$$
$$\Rightarrow x = -\frac{1}{2}$$

atics Equations

pected to solve equations in the form ax^2 + bx + c = 0. There are
be confident with each of them.

Completing the Square

This is an excellent method to use if the expression will not factorise and you haven't got a calculator as it will easily put the answer in surd form. However it can be complicated when the x coefficient is greater than 1.

Examples: solve

$$x^2 + 4x - 2 = 0$$
$$\Rightarrow (x+2)^2 - 2^2 - 2 = 0$$
$$\Rightarrow (x+2)^2 - 6 = 0$$
$$\Rightarrow (x+2)^2 = 6$$
$$\Rightarrow x + 2 = \pm\sqrt{6}$$
$$\Rightarrow x = -2 \pm \sqrt{6}$$

solve

$$2x^2 - 7 = 4x$$
$$\Rightarrow 2x^2 - 4x = 7$$
$$\Rightarrow x^2 - 2x = \frac{7}{2}$$
$$\Rightarrow (x-1)^2 - 1^2 = \frac{7}{2}$$
$$\Rightarrow (x-1)^2 = \frac{9}{2}$$
$$\Rightarrow x - 1 = \pm\sqrt{\frac{9}{2}}$$
$$\Rightarrow x - 1 = \pm\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$
$$\Rightarrow x = 1 \pm \frac{3\sqrt{2}}{2}$$

solve

$$10 = 3x - x^2$$
$$\Rightarrow x^2 - 3x + 10 = 0$$
$$\Rightarrow \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + 10 = 0$$
$$\Rightarrow \left(x - \frac{3}{2}\right)^2 + \frac{31}{4} - 0$$
$$\Rightarrow \left(x - \frac{3}{2}\right)^2 = -\frac{31}{4} < 0$$

So no real roots

Quadratic Formula

If all other methods fail use the quadratic formula!

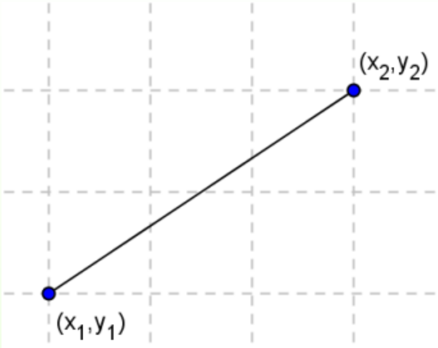
You can also use b^2—4ac to check whether the equation has real roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Straight Line Graphs

Any straight line graph can be put in the form $y=mx+c$, where **m** is the gradient and **c** is the y intercept.

The gradient of a line is found using the formula:



$$m=\frac{y_2-y_1}{x_2-x_1}$$

The length of that line is (by Pythagoras' Theorem)

$$l=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Example: A line is parallel to the line $y=5x+8$ and it goes through $(3,11)$. What is this line's equation?

As the line is parallel to $y=5x+8$ it must have a gradient of 5.
So the line is $y=5x+c$. To find the c value we should substitute in (3,11)

$$\Rightarrow 11=5\times 3+c$$
$$\Rightarrow 11=15+c$$
$$\Rightarrow -4=c$$
$$so \quad y=5x-4$$

Example: Find the equation of the lines that goes through (3,-1) and (6,3). Give your answer in the form: $ax+by+c=0$

$$m=\frac{3-(-1)}{6-3}=\frac{4}{3}$$

$$so\ y=\frac{4}{3}x+c$$

$$Sub\ in\ (6,3)\text{L}\ \Rightarrow 3=\frac{4}{3}\times 6+c$$

$$\Rightarrow 3=8+c \quad \Rightarrow c=-5$$

$$so\ y=\frac{4}{3}x-5 \quad (\times 3)$$

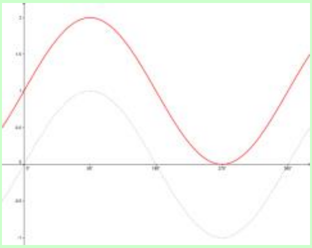
$$\Rightarrow 3y=4x-15$$

$$\Rightarrow 4x-3y-15=0$$

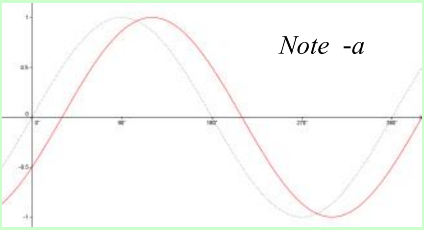
Transformation of graphs

You need to know how to transform and identify transformations of graphs. The rules are summarised below.
The original function f(x) is shown as a dotted curve.
(If you are unfamiliar with the trigonometry graphs then you should revise these).

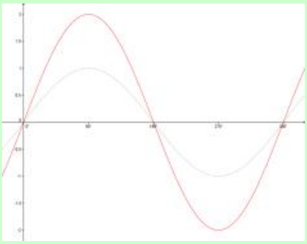
f(x) + a moves the graph of f(x) 'a' units vertically



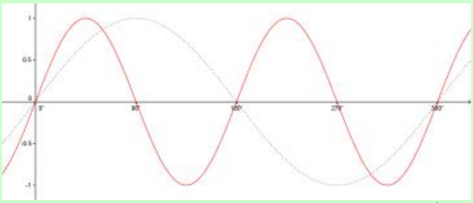
f(x – a) moves the graph of f(x) 'a' units horizontally



af(x) stretches the graph of f(x) 'a' times vertically



f(ax) contracts the graph 'a' times horizontally



Example: Calculate the minimum point of $y = x^2 + 6x + 5$
Complete the square to get: $y = (x+3)^2 - 9 + 5 \Rightarrow y = (x+3)^2 - 4$
$y=x^2$ has a minimum point of
(0,0) so using f(x-a) the x coordinate of the minimum point has moved to x=-3, and using f(x) + a the y coordinate has moved to –4 so the min point is (-3,-4).

Mechanics

Trigon

Mechanics can be thought of as the mat affect an object's speed and direction. If yo help. If not it gives an excellent example of how the advanced maths you A lot of the methods you will use in Mechanics are extensions of GCSE t **trigonometry** and **vectors**.

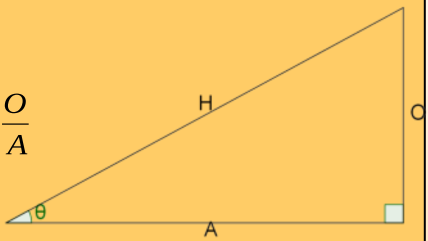
Trigonometry

At GCSE you will have studied trigonometry in right angled and non-right angled triangles. Both techniques are useful for Mechanics. The key formulae are:

Right Angled

$$\sin\theta = \frac{O}{H}, \cos\theta = \frac{A}{H}, \tan\theta = \frac{O}{A}$$

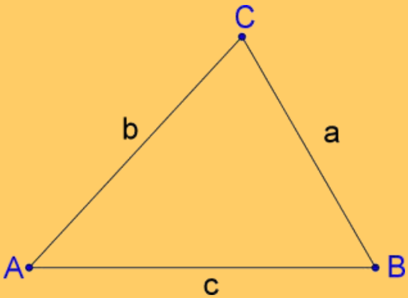
Remember **SOHCAHTOA**



Non-Right Angled

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$



Example: A man walks 7m on a bearing of 050° from his chair at A to an ice cream van at B then 6m on a bearing of 140° to a bin at C. Calculate how far, as the crow flies, he is from his chair.

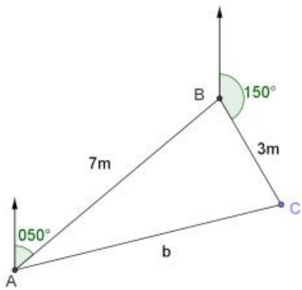
Angle ABC must be 80° and now we can use the cosine rule.

$$b^2 = 3^2 + 7^2 - 2\times3\times7\times\cos 80$$

$$b^2 = 9 + 49 - 42\cos 80$$

$$b^2 = 50.71$$

$$b = 7.12m \quad (3SF)$$



ometry and Travel Graphs

ths of the physical world. In Mechanics you will look at how forces
ou are studying Physics then this part of the course will be a great
ı learn is used in real situations.
echniques. The main methods you need to be comfortable with are

Travel Graphs

All the way through school you will have seen travel graphs. In Mechanics we use **Distance Time** graphs or **Velocity Time** graphs. (Velocity is speed but with a direction)

Key Facts

Distance Time

- The gradient is the velocity
- If the gradient is linear then the velocity is constant
- Flat horizontal lines means no movement

Velocity Time

- The gradient is the acceleration
- If the gradient is linear then the acceleration is constant
- Flat horizontal lines mean a constant velocity
- The area under the graph is the distance travelled.

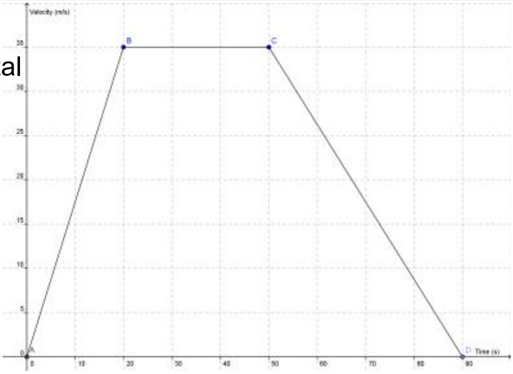
Example: The diagram shows a velocity time graph. Calculate the total distance travelled.

Use the area under the graph

$$= \frac{1}{2}\times 35\times 20 + 30\times 35 + \frac{1}{2}\times 35\times 40$$

$$= 350 + 1050 + 700$$

$$= 2100\text{m} = 2.1\text{km}$$



Statistics at 'A' Level is very different to wl

Cumulative Frequency and Box Plots

As well as plotting both of these types of chart you must be able to read off key values such as the Quartiles and then interpret the result.

Remember when plotting a cumulative frequency curve you plot the upper limit of the class against c. frequency. A box plot shows the minimum, Lower Quartile, Median, Upper Quartile and maximum. value.

Example: Draw a cumulative frequency curve and box plot for the following data on heights of flowers.

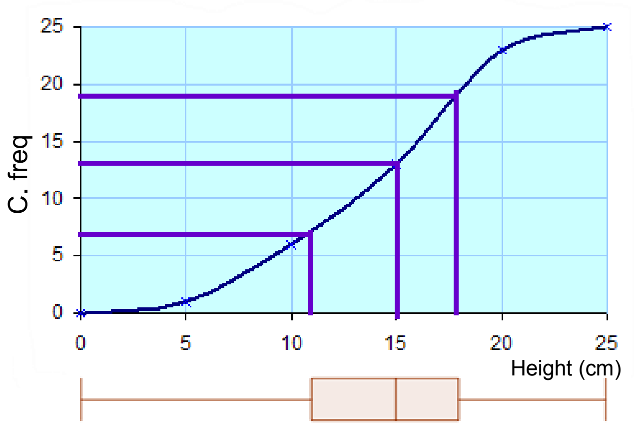
Height (h cm)	Freq	Upper limit	C. Freq
0 < h ≤ 5	1	≤ 5	1
5 < h ≤ 10	5	≤ 10	6
10 < h ≤ 15	7	≤ 15	13
15 < h ≤ 20	10	≤ 20	23
20 < h ≤ 25	2	≤ 25	25

There are 25 values so:

$$Q_1 = \frac{25+1}{4} = 6.5^{th}$$

$$Q_2 = \frac{25+1}{2} = 13^{th}$$

$$Q_3 = \frac{3(25+1)}{4} = 18.5^{th}$$



...ative Frequency and Histograms

...hat you studied at GCSE.

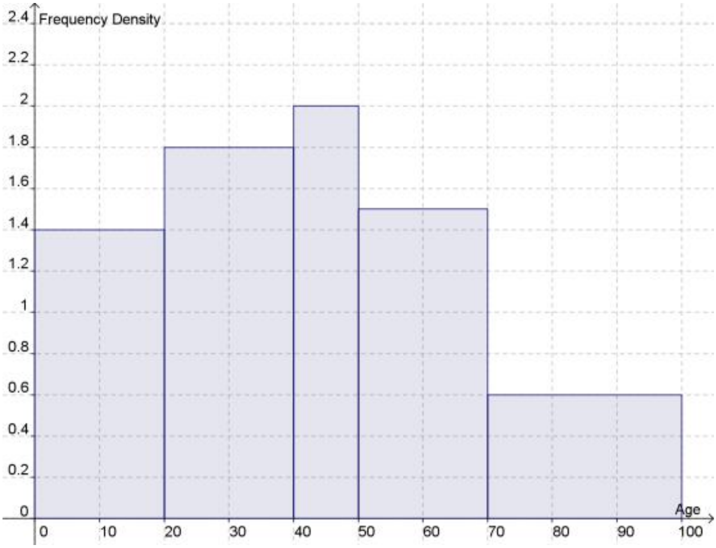
Histograms

Histograms are like bar charts, but have important differences. In a histogram there are no gaps between the bars and the area of the bar is proportional to the frequency. To draw a histogram we use **frequency density** which is

$$fd = \frac{Frequency}{Class\ width}$$

Example: Draw a histogram for the following data..

Age	Freq	cw	fd
0 < h ≤ 20	28	20	28 ÷ 20 = 1.4
20 < h ≤ 40	36	20	36 ÷ 20 = 1.8
40 < h ≤ 50	20	10	20 ÷ 10 = 2
50 < h ≤ 70	30	20	30 ÷ 20 = 1.5
70 < h ≤ 100	18	30	18 ÷ 30 = 0.6



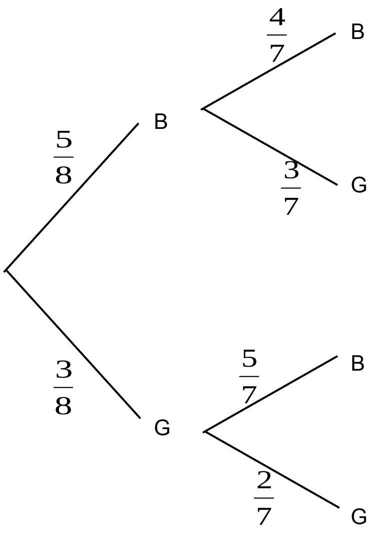
A large part of all the Statistics course is p look at more practical applications, including the odds of winning the lotte have studied for a while.

Probability

You should be familiar with all the basic rules for probability and how to find the probability of two or more events. This might mean using a **sample space diagram** or a **tree diagram**.

Remember that the sum of the probabilities of all possible outcomes is always 1, so P(A') = 1 - P(A).

Example: A bag contains 5 blue marbles and 3 green marbles. A marble is picked out and not replaced. This is then repeated. Draw a tree and work out the probabilities of picking 2 marbles of different colours.



$$\frac{5}{8}\times\frac{4}{7}=\frac{20}{56}=\frac{5}{14}$$

$$\frac{5}{8}\times\frac{3}{7}=\frac{15}{56}$$

$$\frac{3}{8}\times\frac{5}{7}=\frac{15}{56}$$

$$\frac{3}{8}\times\frac{2}{7}=\frac{6}{56}$$

So P(different colours) = P(BG) + P(GB)

$$=\frac{15}{56}+\frac{15}{56}$$

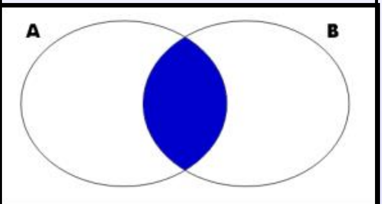
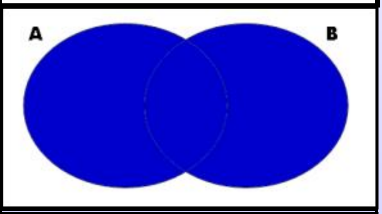
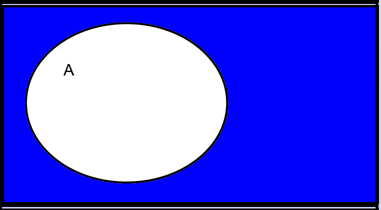
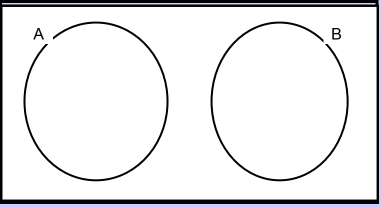
$$=\frac{30}{56}=\frac{15}{28}$$

bility and Venn Diagrams

probability. At A Level you will go into a lot more detail than GCSE and
ery (very small)! You will also need Venn diagrams which you may not

Venn Diagrams

Venn diagrams are used for sorting information. They are also very useful for looking at probabilities as we can see if two events are **mutually exclusive** or not. It is useful to know the correct terminology for the sets on Venn diagrams and its probability equivalent.

Set Theory	Probability	Diagram
$A \cap B$ (A intersect B)	A **and** B occur	
$A \cup B$ (A union B)	A **or** B occur	
A' (Not A)	**Not** A	
A and B are **disjoint**	A and B are **mutually exclusive**	

Decision Maths

Decision Maths is the maths of efficiency and was the part of Maths that gave birth to Computer Science.
In this part of the Further Maths course you will look at how to find quickest routes, find your ideal blind date, sort large lists and refine economic models to create the most profit.

Dijkstra's Algorithm for shortest route

This algorithm is used by Sat Navs to find the route.

1. Label the start vertex with permanent label 0 and order label 1
2. Assign temporary labels to all the vertices that can be reached directly from the start
3. Select the vertex with the smallest temporary label and make its label permanent. Add the correct order label.
4. Put temporary labels on each vertex that can be reached directly from the vertex you have just made permanent. The temporary label must be equal to the sum of the permanent label and the direct distance from it. If there is an existing temporary label at a vertex, it should be replaced only if the new sum is smaller.
5. Select the vertex with the smallest temporary label and make its label permanent. Add the correct order label.
6. Repeat until the finishing vertex has a permanent label.
7. To find the shortest paths(s), trace back from the end vertex to the start vertex. Write the route forwards and state the length.

Watch this YouTube clip for more information.

https://www.youtube.com/watch?v=pVfj6mxhdMw